TURBULENT VISCOSITY COEFFICIENT OF VISCOPLASTIC MEDIA WITH LOW PLASTICITY

A. M. Makarov, L. K. Martinson, and K. B. Pavlov

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It is shown that when a viscoplastic fluid has weakly plastic properties the effective turbulent viscosity of the flow may be lowered.

A recent series of experimental investigations has reliably established the fact that the hydraulic resistance for turbulent flow regimes in channels is lowered by introducing small quantities of additives into the flow [1]. In view of possible practical applications, it is important to explain the nature of this phenomenon. As yet, however, there is no single satisfactory theory to explain it. Some authors [2], for example, attribute the lowering of hydraulic resistance to a decrease of viscosity in the layer next to the channel wall on introducing surface-active additives into the stream. Others [3] explain it as the result of the formation of a distinctive supermolecular structure in the flow, which damps the high-frequency part of the turbulent pulsation spectrum. These two explanations do not exclude each other, however, and it is possible that both causes are operative simultaneously.

The present paper discusses still another explanation for the lowering of hydraulic resistance in turbulent flow regimes by introducing additives into the flow. It is based on the supposition that the supermolecular structure formed by introducing additives to the fluid transforms the fluid into a general class of non-Newtonian media, the so called viscoplastic fluids. These media are considered in rheology, and some examples are oil paints, solutions of clay in water, and pastes [4, 5]. The internal spatial structure is governed by the fact that viscoplastic fluids have a certain limiting shear stress τ_0. For stresses less than τ_0, the structure turns out to be sufficiently rigid not to break down, and the viscoplastic fluid behaves like a solid elastic body. For stresses greater than τ_0, the viscoplastic fluid behaves like an ordinary Newtonian fluid. The rheological equation of a viscoplastic fluid connecting the shear stress τ with the rate of shear deformation $\dot{\gamma} = dv/dy$ has the following form for two-dimensional motion:

$$\tau = \frac{\dot{\gamma}}{|\dot{\gamma}|}\tau_0 + \eta\dot{\gamma}, \qquad (1)$$

where the plastic viscosity η and τ_0 are rheological characteristics of the medium. For $\tau_0 = 0$ we have the case of an ordinary Newtonian fluid.

If the fluids in the experiments [1] really had a supermolecular structure, they could certainly be regarded as viscoplastic with vanishingly small values of τ_0. The presence of a small limiting shear stress τ_0 would be virtually insignificant for laminar flow regimes, but on the other hand could turn out to be important for turbulent flow.

It is well known that in developed turbulence the kinetic energy of the average flow is transferred through pulsations of successively smaller scale sizes. Energy dissipation occurs in the smallest of these, smaller than the Kolmogorov inner scale l [6]. If the medium has viscoplastic properties, we may assume that those pulsations are damped in which values of shear stress τ_0 are not attained. We shall show that, for vanishingly small τ_0, this action may lead to a diminution of the effective turbulent viscosity.

We shall consider developed turbulent motion in a straight cylindrical pipe, where the fluid is viscoplastic with a small value of limiting shear stress ($\tau_0 \to 0$). Assuming that the plastic properties of the fluid are of fundamental importance for the smallest pulsation scale sizes in the interval $\lambda < l$, we shall suppose that all the conclusions of turbulence theory for ordinary Newtonian fluids are valid for the entire inertial interval of pulsation scale sizes $\lambda > l$ and for scale sizes l approximately equal to the inner scale ($\lambda \sim l$). Thus, if U is the maximum value of the velocity at the center of the pipe, and L is the diameter of the pipe, then the energy dissipated per unit time per unit mass is defined as

$$\varepsilon \sim \frac{U^3}{L}, \qquad (2)$$

and the inner scale l and the corresponding pulsation velocity v_l for this scale are

$$l \sim \frac{L}{R^{3/4}}, \qquad (3)$$

$$v_l \sim \frac{U}{R^{1/4}}, \qquad (4)$$

where $R = UL/\nu$ is the Reynolds number of the main flow [6].

In the case of a Newtonian fluid, the corresponding velocities v_λ for pulsations of scale sizes less than the inner scale l may be represented in the form of an expansion in powers of λ, where it is sufficient to confine ourselves to the first term in the first approximation [6]:

$$v_\lambda = a_1\lambda, \quad a_1 = \text{const}. \qquad (5)$$

The order of magnitude of the expansion coefficient a_1 is found from (3)–(4), since $v_\lambda = v_l$ for $\lambda = l$:

$$a_1 \sim \frac{U}{L}R^{1/2} \qquad (6)$$

In the case of a viscoplastic fluid with small τ_0, the expansion in powers of λ in the expression for v_λ (5) should contain a zero-order term a_0, which is a function of the plasticity parameter τ_0:

$$v_\lambda = a_0 + a_1 \lambda, \quad a_0, \ a_1 = \text{const.} \tag{7}$$

As λ decreases, the velocity decreases, and beginning from some scale size s, pulsations with scale sizes $\lambda < s$ will be suppressed due to the plastic properties of the medium. The scale size s of the smallest permissible pulsations may be determined from (7) and the condition $v_s = 0$:

$$s = -\frac{a_0}{a_1}. \tag{8}$$

On the assumption which has been made, the plastic properties of the fluid exert a negligible effect on the velocity pulsations for scale sizes $\lambda \approx l$, in view of the smallness of τ_0. Thus,

$$|a_0| \ll a_1 l \tag{9}$$

or, from (8),

$$s \ll l. \tag{10}$$

We stress that the strong inequality (10), a result of the fact that the medium has only weakly plastic properties, is at the same time the condition for retaining the results of turbulence theory for Newtonian fluids in the inertial interval of scale sizes $\lambda > l$ and for $\lambda \approx l$. Consequently, expression (6) may be retained for determining the coefficient a_1. Together with this, the relation (1) may be applied to pulsations of scale size l, to give

$$\frac{v_l}{l} \approx \frac{\tau_l - \tau_0}{\eta}, \tag{11}$$

where τ_l is the shear stress in pulsations of scale size l. Comparing (11) and (7), for $\lambda = l$ we may write down an expression for the coefficient a_0 in the form

$$a_0 \approx \frac{\tau_0}{\eta} l. \tag{12}$$

As is to be expected, $a_0, s \to 0$ for $\tau_0 \to 0$. Inserting (6) and (12) in (8), we have the following expression for s:

$$s \approx \frac{\tau_0 l}{\eta} \frac{L}{UR^{1/2}} \equiv lkR^{1/2}, \tag{13}$$

where $k \equiv \tau_0/\rho U^2$ is a dimensionless criterion of the plastic properties of the medium. It follows from (10) and (13) that $kR^{1/2} \ll 1$.

Relations (5) and (6) may be obtained in the case of a Newtonian fluid, if we directly equate expression (2) to the quantity

$$\varepsilon \sim \nu \left(\frac{v_\lambda}{\lambda} \right)^2, \tag{14}$$

which determines the energy ε dissipated through the velocity gradient of one of the pulsations of scale size $\lambda < l$ in which the dissipation actually occurs [6].

In the case of a viscoplastic fluid with vanishingly small τ_0, the definition (14) may be retained only for those values of $\lambda \approx l$, for which the presence of plastic properties is unimportant. Thus, setting $v_\lambda = a_0 + a_1\lambda$ in (14) and $\lambda = l$, and neglecting small second-order quantities, we obtain

$$\varepsilon \sim \nu \left(a_1^2 + \frac{2a_0 a_1}{l} \right). \tag{15}$$

Finally, determining the turbulent viscosity ν_T from the expression

$$\varepsilon \sim \nu_t \frac{U^2}{L^2} \tag{16}$$

and equating (15) and (16), while taking (6) and (12) into account, we obtain

$$\nu_t \sim \nu [R - 2kR^{3/2}]. \tag{17}$$

Thus, on increasing the plasticity criterion k, the effective turbulent viscosity may decrease. It should, however, be remembered that this proof refers only to viscoplastic media with vanishingly small τ_0, for which $kR^{1/2} \ll 1$.

NOTATION

a_0 and a_1 are pulsation-velocity expansion coefficients; v_λ, v_l, and v_s are pulsation velocities; τ and τ_l are shear stresses; τ_0 is the limiting shear stress; γ is the rate of shear deformation; l is the inner scale size of the turbulence; s is the scale size of the minimum permissible pulsations; L is the diameter of the pipe; U is the velocity at the center of the pipe; ν is the kinematic viscosity coefficient; ν_t is the turbulent viscosity coefficient; R is the Reynolds number for the main flow; k is the dimensionless plasticity criterion; ε is the dissipated-energy density.

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Bauman Higher Technical School, Moscow